

# UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

#### **ADDITIONAL MATHEMATICS**

4037/11

Paper 1 May/June 2013

2 hours

Candidates answer on the Question Paper.

No additional materials are required.

#### **READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

#### Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

This document consists of 16 printed pages.



### Mathematical Formulae

### 1. ALGEBRA

Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \ .$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where *n* is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ .

#### 2. TRIGONOMETRY

*Identities* 

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\csc^2 A = 1 + \cot^2 A$$

Formulae for  $\triangle ABC$ 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

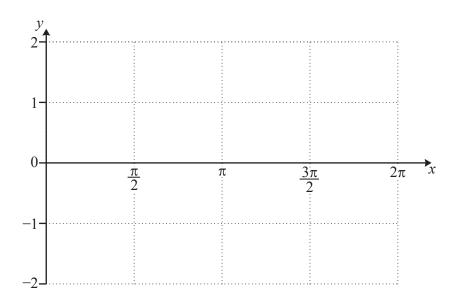
1 On the axes below sketch, for  $0 \le x \le 2\pi$ , the graph of



(i) 
$$y = \cos x - 1$$
,

[2]

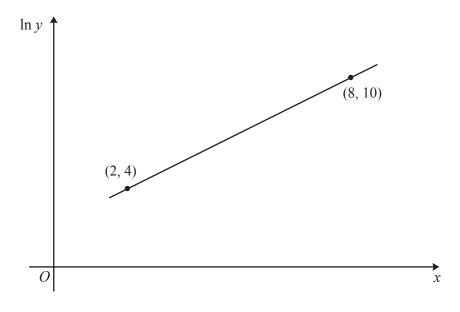
(ii)  $y = \sin 2x$ .



(iii) State the number of solutions of the equation  $\cos x - \sin 2x = 1$ , for  $0 \le x \le 2\pi$ . [1]

Variables x and y are such that  $y = Ab^x$ , where A and b are constants. The diagram shows the graph of  $\ln y$  against x, passing through the points (2, 4) and (8, 10).

For Examiner's Use



Find the value of A and of b.

[5]

3	A committee of 6 members is to be selected from 5 men and 9 women. Find the number of different committees that could be selected if								
	(i)	there are no restrictions,	[1]	Use					
	(ii)	there are exactly 3 men and 3 women on the committee,	[2]						
	(iii)	there is at least 1 man on the committee.	[3]						

4 (i) Given that  $\log_4 x = \frac{1}{2}$ , find the value of x.

[1] For Examiner's Use

(ii) Solve  $2\log_4 y - \log_4 (5y - 12) = \frac{1}{2}$ .

[4]

5 (i) Find 
$$\int \left(1 - \frac{6}{x^2}\right) dx$$
.

(ii) Hence find the value of the positive constant 
$$k$$
 for which  $\int_{k}^{3k} \left(1 - \frac{6}{x^2}\right) dx = 2$ . [4]

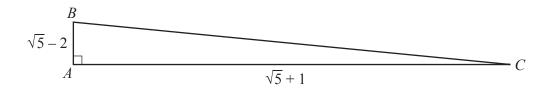
6 (i) Given that 
$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$$
, find  $\mathbf{A}^{-1}$ .

(ii) Using your answer from part (i), or otherwise, find the values of a, b, c and d such that

$$\mathbf{A} \begin{pmatrix} a & b \\ c & -1 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 17 & d \end{pmatrix}.$$
 [5]

## 7 Calculators must not be used in this question.

For Examiner's Use

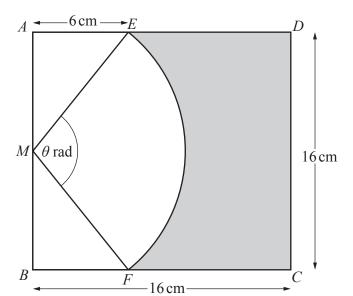


The diagram shows a triangle ABC in which angle  $A = 90^{\circ}$ . Sides AB and AC are  $\sqrt{5} - 2$  and  $\sqrt{5} + 1$  respectively. Find

(i)  $\tan B$  in the form  $a + b\sqrt{5}$ , where a and b are integers, [3]

(ii)  $\sec^2 B$  in the form  $c + d\sqrt{5}$ , where c and d are integers. [4]

8



For Examiner's Use

The diagram shows a square ABCD of side 16 cm. M is the mid-point of AB. The points E and F are on AD and BC respectively such that AE = BF = 6 cm. EF is an arc of the circle centre M, such that angle EMF is  $\theta$  radians.

(i) Show that  $\theta = 1.855$  radians, correct to 3 decimal places. [2]

(ii) Calculate the perimeter of the shaded region.

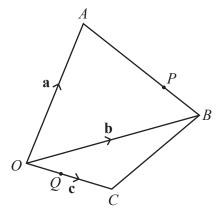
[4]

(iii) Calculate the area of the shaded region.

For Examiner's Use

[3]

9



For Examiner's Use

The figure shows points A, B and C with position vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  respectively, relative to an origin O. The point P lies on AB such that AP:AB=3:4. The point Q lies on OC such that OQ:QC=2:3.

(i) Express  $\overrightarrow{AP}$  in terms of **a** and **b** and hence show that  $\overrightarrow{OP} = \frac{1}{4}(\mathbf{a} + 3\mathbf{b})$ . [3]

(ii) Find  $\overrightarrow{PQ}$  in terms of  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$ . [3]

(iii) Given that  $5\overrightarrow{PQ} = 6\overrightarrow{BC}$ , find **c** in terms of **a** and **b**.

[2] For Examiner's Use

10 The point A, whose x-coordinate is 2, lies on the curve with equation  $y = x^3 - 4x^2 + x + 1$ .

For Examiner's Use

[4]

(i) Find the equation of the tangent to the curve at A.

This tangent meets the curve again at the point B.

(ii) Find the coordinates of B.

[4]

(iii) Find the equation of the perpendicular bisector of the line AB. [4] For Examiner's UseQuestion 11 is printed on the next page.

11 (a) Solve 
$$2\sin\left(x + \frac{\pi}{3}\right) = -1$$
 for  $0 \le x \le 2\pi$  radians.

**(b)** Solve 
$$\tan y - 2 = \cot y$$
 for  $0^{\circ} \le y \le 180^{\circ}$ .

[6]

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